I hope, it is entertaining as an April 1 eprint should be, but in fact it is about serious things.

--Z.K. Silagadze

Brainwashed by Newton?

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Abstract

At first sight, arguments for and against of the relativistic mass notion look like a notorious intra-Lilliputian quarrel between Big-Endians (those who broke their eggs at the larger end) and Little-Endians. However, at closer inspection we discover that the relativistic mass notion hinders understanding of the spirit of modern physics to a much greater extent than it seems.

1 Introduction

Velocity dependent relativistic mass is still popular in teaching of relativity and especially in popular literature. Several authors have criticized the use of this historically outdated notion [1, 2, 3, 4, 5, 6, 7] while others find the concept useful [8, 9, 10, 11, 12].

The usage of relativistic mass and "Einstein's most famous equation $E = mc^2$" [13] was quite ubiquitous in old textbooks. So why should we bother? If such renowned experts in the field as Tolman, Born and Fock in the past and Penrose and Rindler today find the concept of relativistic mass useful why not to follow the motto "All true believers break their eggs at the convenient end" [14] instead of entering in an endless and arid dispute between Big-Endians and Little-Endians?

The answer is simple. Modern physics offers a picture of reality which is completely different from the classical Newtonian picture. It is impossible to master this kind of reality if you try to put it in a Procrustean bed of Newtonian concepts. Nevertheless this is exactly what the modern education is trying to do. And not only in the realm of special relativity. “Most elementary textbooks and popularization works about quantum physics remain plagued by archaic wordings and formulations” [15].

Modern civilization depends on advances in science more than ever before. On the other hand the current practice of teaching does not cultivate the conceptual critical thinking skills and is still oriented on the authoritarian teaching traditions. Archaic concepts in teaching obscure vision of the world offered by modern physics. “Upon failure to develop this vision, necessarily critical, we can find ourselves in a world of machines, both physical and intellectual, that would work fairly well, but we would not understand them any more. This means that the progress of science is not guaranteed” [16].

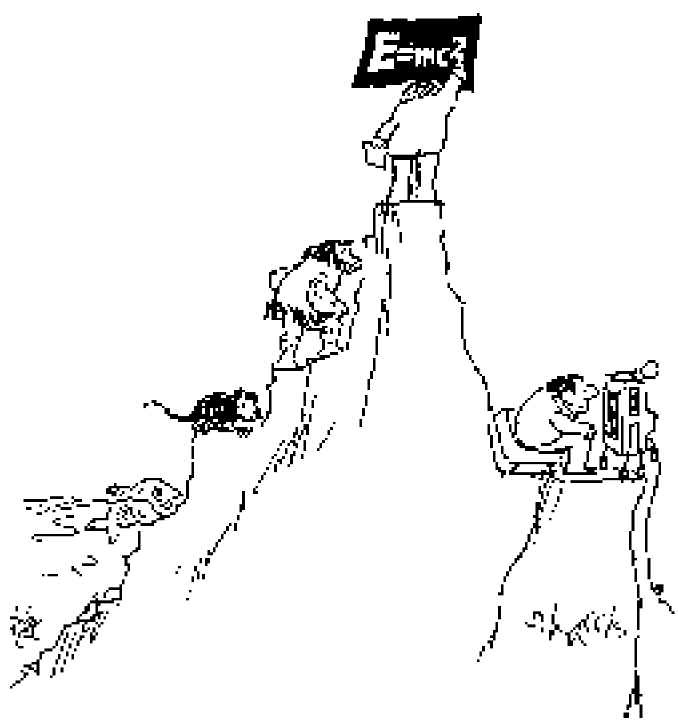

Figure 1: Scientific progress is not guaranteed. Computers and Internet not necessarily lead to higher level of progress if scientific knowledge is not disseminated (the figure is from [16]).

The concept of mass in modern physics is quite different from the Newtonian concept of mass as a measure of inertia. However, this does not mean that we should throw out mass as a measure of inertia. Simply modern physics framework is more general and flexible and it explicitly indicates the context under which it is fairly safe to consider the mass as a measure of inertia. The problems begin when things are turned upside down and the Newtonian physics is considered as a basic truth and modern physics as some derivative from it. "Objectivity of Classical physics is some sort of half-truth. It is a very good thing, a very great achievement, but somehow it makes it more difficult than it would have seemed before to understand the fullness of reality" [17].

In this note we outline the way how the concept of mass is introduced in the modern physics and make it clear that this way leaves no room for relativistic mass. This archaic concept is doomed to loose its role as a portal in relativistic world if we want new generations to fully appreciate the benefits of the twentieth-century scientific revolution.

Of course, no disrespect is implied by the title. It is a remake of the title of Philip Anderson's famous article "Brainwashed by Feynman?" [18] and our goal of using such a title is the same: just to sharpen reader's attention to a real problem. The problem here is that modern education lags far behind the frontier of modern physics.

2 Landau & Lifshitz way of introducing mass

Okun remarks [19] that the first textbook in the world in which mass was velocity-independent was "The classical theory of fields" by Landau and Lifshitz, first published in 1940. There was a good reason why Landau and Lifshitz did not use relativistic mass: they had based their presentation on the principle of least action. And this method leaves little room for relativistic mass, or, rather, makes its use obsolete and unnecessary.

Indeed, let us consider a free relativistic particle. Landau and Lifshitz's reasoning goes as follows [20]. The action integral for this particle should be independent on our choice of reference frame, according to the principle of relativity. Furthermore, the integrand should be a differential of the first order. A free particle can provide only one scalar of this kind - the interval $ds = \sqrt{c^2\, dt^2 - d\vec{r}^{\,2}}$. Therefore, for a free particle the expected form of the action is

$$S = -\alpha \int\limits_a^b ds = -\alpha c \int\limits_{t_1}^{t_2} \sqrt{1 - \frac{v^2}{c^2}}\, dt. \tag{1}$$

Here c is the light velocity and the first integral is along the world line of the particle, a and b being two particular events of the arrival of the particle at the initial and final positions at definite times t_1 and t_2. α is some constant and it must be positive lest the action become unbounded from below.

The physical meaning of the constant α becomes evident if we consider the non-relativistic limit of the relativistic Lagrangian from (1):

$$L = -\alpha c \sqrt{1 - \frac{v^2}{c^2}} \approx \alpha\, \frac{v^2}{2c} - \alpha c. \tag{2}$$

This is equivalent to the non-relativistic Lagrangian $L = mv^2/2$ if and only if $\alpha = mc$.

The way how the mass m appeared in the relativistic Lagrangian indicates clearly that it is an invariant quantity and does not depend on velocity. But how about $p = mv$ which is notoriously used to introduce relativistic mass?

Simply $p = mv$ is a wrong way to define momentum. It is only true in non-relativistic situations and is no longer valid when the particle velocity approaches the velocity of light c.

The modern way to introduce momentum is the Noether theorem which relates symmetries of a theory with its laws of conservation. In relativity, it is better to deal with the time t and spatial coordinates x_i on equal footing. Therefore, we introduce a parametrization of the particle's worldline

$$x_i = x_i(\tau), \quad t = t(\tau),$$

where τ is some (scalar) evolution parameter, and rewrite the action of a dynamical system as follows

$$S = \int\limits_{t_1}^{t_2} L(x_i, v_i, t)\, dt = \int\limits_{\tau_1}^{\tau_2} L\left(x_i, \frac{\dot{x}_i}{\dot{t}}, t\right) \dot{t}\, d\tau = \int\limits_{\tau_1}^{\tau_2} \mathcal{L}(x_i, t, \dot{x}_i, \dot{t})\, d\tau, \tag{3}$$

where

$$\dot{x}_i = \frac{dx_i}{d\tau}, \quad \dot{t} = \frac{dt}{d\tau}, \quad v_i = \frac{dx_i}{dt} = \frac{\dot{x}_i}{\dot{t}} \text{ and } \mathcal{L} = L\left(x_i, \frac{\dot{x}_i}{\dot{t}}, t\right) \dot{t}. \tag{4}$$

A symmetry of the action (3) is a transformation

$$x_i \to x_i' = x_i + \delta x_i, \quad t \to t' = t + \delta t, \tag{5}$$

under which the variation of the Lagrangian can be written as a total derivative of some function $F(x_i, t, \tau)$ with respect to the evolution parameter τ,

$$\delta \mathcal{L} = \frac{dF}{d\tau}. \tag{6}$$

If a dynamical system with action (3) has a symmetry defined by (5) and (6), then the Noether's current (Einstein summation convention is assumed)

$$J = \frac{\partial \mathcal{L}}{\partial \dot{x}_i}\, \delta x_i + \frac{\partial \mathcal{L}}{\partial \dot{t}}\, \delta t - F \tag{7}$$

is conserved. Indeed, using the Euler-Lagrange equations, which follow from the principle of least action $\delta S = 0$,

$$\frac{d}{d\tau}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) = \frac{\partial \mathcal{L}}{\partial x_i}, \quad \frac{d}{d\tau}\left(\frac{\partial \mathcal{L}}{\partial \dot{t}}\right) = \frac{\partial \mathcal{L}}{\partial t},$$

we get in light of (6)

$$\frac{dJ}{d\tau} = \frac{\partial \mathcal{L}}{\partial x_i}\delta x_i + \frac{\partial \mathcal{L}}{\partial t}\delta t + \frac{\partial \mathcal{L}}{\partial \dot{x}_i}\delta \dot{x}_i + \frac{\partial \mathcal{L}}{\partial \dot{t}}\delta \dot{t} - \frac{dF}{d\tau} = \delta \mathcal{L} - \delta \mathcal{L} = 0.$$

Sometimes it is more convenient to express the conserved Noether current in terms of the original Lagrangian L. Since,

$$\mathcal{L}(x_i, t, \dot{x}_i, \dot{t}) = L(x_i, v_i, t)\, \dot{t},$$

where

$$v_i = \frac{\dot{x}_i}{\dot{t}},$$

we have

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_i} = \dot{t}\, \frac{\partial L}{\partial v_j}\, \frac{\partial}{\partial \dot{x}_i}\left(\frac{\dot{x}_j}{\dot{t}}\right) = \frac{\partial L}{\partial v_i}$$

and

$$\frac{\partial \mathcal{L}}{\partial \dot{t}} = L + \dot{t}\, \frac{\partial L}{\partial v_i}\, \frac{\partial}{\partial \dot{t}}\left(\frac{\dot{x}_i}{\dot{t}}\right) = L - \frac{\partial L}{\partial v_i}\, \frac{\dot{x}_i}{\dot{t}} = L - v_i\, \frac{\partial L}{\partial v_i}.$$

Therefore, in terms of the original Lagrangian L, the Noether current takes the form [21]

$$J = \frac{\partial L}{\partial v_i}\,\delta x_i - \left(v_i\,\frac{\partial L}{\partial v_i} - L\right)\delta t - F = p_i\,\delta x_i - H\,\delta t - F, \tag{8}$$

where

$$p_i = \frac{\partial L}{\partial v_i} \quad \text{and} \quad H = v_i\,\frac{\partial L}{\partial v_i} - L \tag{9}$$

are the desired general definitions of momentum and energy (Hamiltonian) of the dynamical system (for non-relativistic Lagrangian $L = m v_i v_i/2$, the momentum takes its standard form $p_i = m v_i$ and $H = m v_i v_i/2 = p_i p_i/(2m)$ is the kinetic energy).

It is now clear that the symmetry of the free relativistic particle action (1) with respect to the infinitesimal space translations,

$$x_i' = x_i + \epsilon_i, \quad t' = t, \quad F = 0$$

leads to the momentum conservation, while the symmetry with respect to the infinitesimal time translation,

$$x_i' = x_i \quad t' = t + \epsilon, \quad F = 0$$

implies the energy conservation.

From (9), using the relativistic Lagrangian

$$L = -mc^2\sqrt{1 - \frac{\vec{v}^2}{c^2}}, \tag{10}$$

we get the relativistic expressions for the energy and momentum

$$E = \frac{mc^2}{\sqrt{1 - \frac{\vec{v}^2}{c^2}}}, \quad \vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{\vec{v}^2}{c^2}}}. \tag{11}$$

It follows from these equations that

$$\frac{E^2}{c^2} - \vec{p}^2 = m^2 c^2. \tag{12}$$

The equation (12) expresses the most important relativistic facet of mass: for every free particle its energy-momentum four-vector has a fixed magnitude mc.

An interesting question, for some unknown reasons not usually discussed in most classical mechanics textbooks, is what conserved quantity corresponds to Lorentzian (and Galilean) boosts [22, 23, 24, 25, 26]. An infinitesimal Lorentz boost in the x-direction

$$t' = t - \frac{\epsilon}{c^2}x, \quad x' = x - \epsilon t, \quad y' = y, \quad z' = z$$

is the symmetry of the action (3) with $F = 0$. Therefore, the corresponding Noether current implies the conserved quantity

$$p_x t - \frac{E}{c^2}x = \text{const},$$

or in the vector form after the invariance with regard to the other two boosts are also taken into account

$$\vec{p}\,t - \frac{E}{c^2}\,\vec{r} = \text{const}. \tag{13}$$

We get in fact a relativistic version of the Newton's first law that a free particle moves uniformly with constant velocity

$$\vec{v} = \frac{\vec{p}c^2}{E}. \tag{14}$$

"You aren't used to calling this a conservation law, but it is, and in fact it is the Lorentz partner of the angular momentum conservation law" [25].

3 Mass, cocycles and central extensions

Landau and Lifshitz's argument is seductive but only half true. Let's see how they get the non-relativistic free particle Lagrangian [27]. Homogeneity of space and time implies that L must be independent of $\vec{r}$ and t, so it is a function of the particle's velocity $\vec{v}$ and in fact a function of its magnitude only because the space is isotropic. Under the infinitesimal Galilei transformations

$$x_i' = x_i - \epsilon_i t, \quad t' = t, \tag{15}$$

the Lagrangian $L(v^2)$ can get, at most, a variation which is a total time derivative of some function of coordinates and time (that is Galilean boosts are symmetries of the corresponding action with possibly nonzero F). But under the Galilei boost (15)

$$v_i' = v_i - \epsilon_i$$

and

$$\delta L = \frac{dL}{dv^2} 2v_i \delta v_i = -2v_i \epsilon_i \frac{dL}{dv^2}.$$

This is a total time derivative if and only if $\frac{dL}{dv^2}$ is a constant. Therefore, we can write the Lagrangian as follows

$$L = \frac{m}{2}\vec{v}^2. \tag{16}$$

Once again, the derivation makes it clear that the mass m is a Galilean invariant quantity independent of velocity.

However, unlike the relativistic case, only quasi-invariance of the Lagrangian is required under Galilei transformations (that the variation of the Lagrangian should be a total time derivative). Why such a difference? It is not that we can not find the Lagrangian which is invariant under Galilean transformations (15). We can. Just adding a total time derivative to the Lagrangian (16) we get the Lagrangian $\tilde{L}$ which is evidently invariant under the Galilean boosts [26]:

$$\tilde{L} = L + \frac{d}{dt}\left(-\frac{m\vec{r}^2}{2t}\right) = \frac{m}{2}\left(\vec{v} - \frac{\vec{r}}{t}\right)^2, \tag{17}$$

Of course, $\tilde{L}$ explicitly depends on $\vec{r}$ and t. But this fact, contrary to what is claimed in [27], does not mean a violation of space-time homogeneity. Under space-time translations

$$t' = t + \tau, \quad \vec{r}' = \vec{r} + \vec{a}$$

the variation of $\tilde{L}$ is

$$\delta\tilde{L} = \frac{d}{dt}\left[\frac{m}{2}\left(\frac{\vec{r}^2}{t} - \frac{(\vec{r}+\vec{a})^2}{t+\tau}\right)\right].$$

Therefore, the Lagrangian $\tilde{L}$ is quasi-invariant under space-time translations and this is sufficient to ensure space-time homogeneity.

As we see, Landau and Lifshitz's logic, while obtaining free particle Lagrangians in [20] and [27], contains loopholes. To close these loopholes, more thorough investigation is needed [28] (see also [26, 29, 30]).

For notational simplicity, let q denotes a space-time point $(t(s), \vec{r}(s))$, and let the Lagrangian $\mathcal{L}(q, \dot{q})$ be quasi-invariant with respect to the symmetry group G. That is for any symmetry transformation $g \in G$, we have

$$\mathcal{L}(gq, g\dot{q}) = \mathcal{L}(q, \dot{q}) + \frac{d}{ds}\alpha(g; q). \tag{18}$$

The action

$$S(q_1, q_2) = \int\limits_{s_1}^{s_2} \mathcal{L}(q, \dot{q})\, ds, \tag{19}$$

considered as a function of the trajectory end-points, transforms as follows

$$S(gq_1, gq_2) = S(q_1, q_2) + \alpha(g; q_2) - \alpha(g; q_1). \tag{20}$$

Lévy-Leblond calls $\alpha(g; q)$ a gauge function. If this function has the form

$$\alpha(g; q) = \phi(q) - \phi(gq) + \chi(g), \tag{21}$$

with some functions ϕ and χ, then we can choose a new equivalent action

$$\tilde{S}(q_1, q_2) = S(q_1, q_2) + \phi(q_2) - \phi(q_1)$$

which will be invariant under all symmetry transformations from G (this follows simply from (20) and (21))

$$\tilde{S}(gq_1, gq_2) = \tilde{S}(q_1, q_2).$$

In this case the gauge function $\alpha(g; q)$ is said to be equivalent to zero. Of course, two gauge functions are essentially the same (are equivalent) if their difference is equivalent to zero. It is, therefore, convenient to fix the gauge and choose one representative from each equivalence class with the property

$$\alpha(g; q_0) = 0 \quad \text{for any} \quad g \in G, \tag{22}$$

where q_0 denotes a conventional origin $\vec{r} = 0$, $t = 0$ in space-time (of course, any point can be chosen as the origin, due to space-time homogeneity). Such a representative always exists because if $\alpha(g; q_0) \neq 0$, we choose as an representative an equivalent gauge function $\tilde{\alpha}(g;q) = \alpha(g;q) - \chi(g)$, with $\chi(g) = \alpha(g;q_0)$.

The gauge functions have the following important property [28]. The compatibility of

$$S(g_1g_2q_1,\ g_1g_2q_2) = S(q_1,q_2) + \alpha(g_1g_2;q_2) - \alpha(g_1g_2;q_1)$$

and

$$S(g_1g_2q_1,\ g_1g_2q_2) = S(g_2q_1,\ g_2q_2) + \alpha(g_1;g_2q_2) - \alpha(g_1;g_2q_1) =$$
$$S(q_1,q_2) + \alpha(g_2;q_2) - \alpha(g_2;q_1) + \alpha(g_1;g_2q_2) - \alpha(g_1;g_2q_1)$$

requires

$$\alpha(g_2;q_1) + \alpha(g_1;g_2q_1) - \alpha(g_1g_2;q_1) = \alpha(g_2;q_2) + \alpha(g_1;g_2q_2) - \alpha(g_1g_2;q_2).$$

That is the function

$$\xi(g_1,g_2) = \alpha(g_2;q) + \alpha(g_1;g_2q) - \alpha(g_1g_2;q) \tag{23}$$

does not depends on the space-time point q.

Some elementary cohomology terminology will be useful at this point [31]. Any real function $\alpha_n(g_1,g_2,\ldots,g_n;q)$ will be called a cochain. The action of the coboundary operator δ on this n-cochain produces $(n+1)$-cochain and is determined as follows

$$\begin{gathered}(\delta\alpha_n)(g_1,g_2,\ldots,g_n,g_{n+1};q) = \\ \alpha_n(g_2,g_3,\ldots,g_n,g_{n+1};g_1^{-1}q) - \alpha_n(g_1\cdot g_2,g_3,\ldots,g_n,g_{n+1};q) + \\ \alpha_n(g_1,g_2\cdot g_3,g_4,\ldots,g_n,g_{n+1};q) - \alpha_n(g_1,g_2,g_3\cdot g_4,g_5,\ldots,g_n,g_{n+1};q) + \\ \cdots + (-1)^n\alpha_n(g_1,g_2,\ldots,g_n\cdot g_{n+1};q) + (-1)^{n+1}\alpha_n(g_1,g_2,\ldots,g_n;q).\end{gathered} \tag{24}$$

The coboundary operator has the following important property

$$\delta^2 = 0. \tag{25}$$

A cochain with zero coboundary is called a cocycle. Because of (25), every coboundary $\alpha_n = \delta\alpha_{n-1}$ is a cocycle. However, not all cocycles can be represented as coboundaries. Such cocycles will be called nontrivial.

In fact $\xi(g_1,g_2)$ defined by (23) is a cocycle. Indeed, according to (24),

$$(\delta\xi)(g_1,g_2,g_3;q) = \xi(g_2,g_3) - \xi(g_1g_2,g_3) + \xi(g_1,g_2g_3) - \xi(g_1,g_2).$$

Substituting (23) into the first three terms, we get after some cancellations

$$\xi(g_2, g_3) - \xi(g_1g_2, g_3) + \xi(g_1, g_2g_3) = \alpha(g_2; g_3q) + \alpha(g_1; g_2g_3q) - \alpha(g_1g_2; g_3q),$$

but this is just $\xi(g_1, g_2)$, as the formula (23) is valid for any space-time point q, and in particular for the point $p = g_3q$. Therefore, $(\delta\xi)(g_1, g_2, g_3; q) = 0$ and $\xi(g_1, g_2)$ is a global (independent on a space-time point q) cocycle.

In the following we will assume the gauge fixing (22). Then (23) with $q = q_0$ gives

$$\xi(g_1, g_2) = \alpha(g_1; g_2q_0). \tag{26}$$

From this relation the following two properties of the admissible cocycles follow. First, if the gauge function $\alpha(g; q)$ is equivalent to zero, then $\xi(g_1, g_2)$ is a trivial cocycle. Indeed, let

$$\alpha(g; q) = \phi(q) - \phi(gq) + \chi(g),$$

Then $\alpha(g; q_0) = 0$ condition gives

$$\chi(g) = \phi(gq_0) - \phi(q_0)$$

and, therefore,

$$\xi(g_1, g_2) = \phi(g_2q_0) - \phi(g_1g_2q_0) + \phi(g_1q_0) - \phi(q_0). \tag{27}$$

On the other hand, if we take (a global) 1-cochain $\beta(g) = \phi(gq_0) - \phi(q_0)$, then its coboundary

$$(\delta\beta)(g_1, g_2) = \beta(g_2) - \beta(g_1g_2) + \beta(g_1)$$

just coincides with the r.h.s. of (27). Therefore, $\xi = \delta\beta$ and hence it is a trivial cocycle.

The second property of the admissible cocycle is that if $h \in \Gamma$ belongs to the stabilizer Γ of the point q_0, so that $hq_0 = q_0$, then $\xi(g, h) = 0$ for all $g \in G$. Indeed, this is evident from (26) and our gauge fixing condition (22).

If the symmetry group G acts transitively on space-time (in fact, in this case the space-time can be identified with the homogeneous space G/Γ [28]), then for any point q there exists such a symmetry g_q that

$$q = g_qq_0. \tag{28}$$

Let $\xi(g_1, g_2)$ be some admissible cocycle, that is such that $\xi(g,h) = 0$ for all $g \in G$ and $h \in \Gamma$. Then the formula

$$\alpha(g;q) = \xi(g, g_q) \tag{29}$$

defines a gauge function such that $\alpha(g;q_0) = 0$. Indeed, first of all $\alpha(g;q)$ is defined by (29) for admissible cocycles uniquely despite the fact that g_q is defined by (28) only up to stabilizer transformation, for any $h \in \Gamma$ we have

$$\xi(g_1, g_2 h) = (\delta\xi)(g_1, g_2, h) + \xi(g_1, g_2) + \xi(g_1 g_2, h) - \xi(g_2, h) = \xi(g_1, g_2).$$

Then, using $g_{gq} = g g_q h$ for some $h \in \Gamma$, we can easily check that

$$\alpha(g_2;q) + \alpha(g_1; g_2 q) - \alpha(g_1 g_2; q) = (\delta\xi)(g_1, g_2, g_q) + \xi(g_1, g_2) = \xi(g_1, g_2).$$

The only question that remains is whether the equivalent admissible cocycles can lead to nonequivalent gauge functions. The answer, in general, turns out to be affirmative [28].

Let $\xi'(g_1, g_2)$ and $\xi(g_1, g_2)$ are two equivalent admissible cocycles, so that

$$\xi'(g_1, g_2) = \xi(g_1, g_2) + \zeta(g_2) - \zeta(g_1 g_2) + \zeta(g_1).$$

The admissibility conditions $\xi'(g,h) = \xi(g,h) = 0$, if $h \in \Gamma$, produces a restriction on the cochain $\zeta(g)$:

$$\zeta(gh) = \zeta(g) + \zeta(h), \quad \text{for any } \ g \in G \ \text{ and } \ h \in \Gamma. \tag{30}$$

In particular, (30) shows that $h \to \zeta(h)$ is a one-dimensional representation of the subgroup Γ.

The gauge functions $\alpha'(g;q)$ and $\alpha(g;q)$ defined by these cocycles are related as follows

$$\alpha'(g;q) = \alpha(g;q) + \zeta(g) + \zeta(g_q) - \zeta(g g_q).$$

Note that $g g_q = g_{gq} h$ with some $h \in \Gamma$ (in fact, $h = g_{gq}^{-1} g g_q$). Therefore, in light of (30), we have

$$\alpha'(g;q) = \alpha(g;q) + \zeta(g) + \zeta(g_q) - \zeta(g_{gq}) - \zeta(h),$$

or

$$\alpha'(g;q) = \alpha(g;q) - \zeta(g_{gq}^{-1} g g_q) + \phi(q) - \phi(gq) + \chi(g),$$

with $\phi(q) = \zeta(g_q)$ and $\chi(g) = \zeta(g)$. As we see, $\alpha'(g;q)$ is equivalent to the gauge function

$$\tilde{\alpha}(g;q) = \alpha(g;q) - \zeta(g_{gq}^{-1} g g_q) = \xi(g, g_q) - \zeta(g_{gq}^{-1} g g_q). \tag{31}$$

Suppose the representation ζ of Γ can be extended to the representation ω of the whole group G. Then we will have

$$\zeta(g_{gq}^{-1} g g_q) = \omega(g_{gq}^{-1} g g_q) = \omega(g) + \omega(g_q) - \omega(g_{gq}),$$

and

$$\alpha(g;q) = \tilde{\alpha}(g;q) + \phi(q) - \phi(gq) + \chi(g),$$

with $\phi(q) = \omega(g_q)$ and $\chi(g) = \omega(g)$. Therefore, α and $\tilde{\alpha}$ are equivalent.

However, if the representation ζ can not be extended on G, then $\tilde{\alpha}(g;q)$ and $\alpha(g;q)$ gauge functions are essentially different (not equivalent).

In fact, formula (31) makes it possible to explicitly construct all different gauge functions related to the symmetry group G. All what is needed is to find all non-trivial two-cocycles of G and all non-trivial one-dimensional representations of the stabilizer subgroup Γ which can not be extended on G [28].

In relativistic classical mechanics, the symmetry group G is the Poincaré group and the stabilizer subgroup Γ is the (homogeneous) Lorentz group. However, the Poincaré group has no non-trivial two-cocycles [32, 33] and the Lorentz group has no non-trivial one-dimensional representations. Therefore, all gauge functions, related to the Poincaré group are equivalent to zero and we conclude that it was quite safe for Landau and Lifshitz to assume strictly invariant relativistic action integral.

In non-relativistic case matters are somewhat more complicated. Now G is the Galilei group with elements $g = (\tau, \vec{a}, \vec{v}, R)$ and it acts on the space-time points $q = (t, \vec{r})$ as follows

$$gq = (t + \tau,\ R\vec{r} - \vec{v}t + \vec{a}), \tag{32}$$

where R symbolically denotes the rotation matrix. The stabilizer subgroup Γ is the homogeneous Galilei group with elements $g = (0, 0, \vec{v}, R)$. As in the relativistic case, Γ has no non-trivial one-dimensional representations. However, the full Galilei group G has a non-trivial two-cocycle discovered by Bargmann [32]. Bargmann's cocycle may be chosen in the form [28]

$$\xi(g_1, g_2) = m\left(\frac{1}{2}\vec{v}_1^{\,2}\,\tau_2 - \vec{v}_1 \cdot R_1\vec{a}_2\right), \tag{33}$$

where m is an arbitrary real number.

Possible gauge functions for the Galilei group are uniquely specified by the equivalence classes of the Bargmann cocycle that is by the number m. We can take $g_q = (t, \vec{r}, 0, 1)$, because $g_q q_0 = q$ with $q_0 = (0, 0)$ and $q = (t, \vec{r})$. Therefore, in accordance with (31), we obtain

$$\alpha(g; q) = \xi(g, g_q) = m\left(\frac{1}{2}v^2 t - \vec{v} \cdot R\vec{r}\right) \tag{34}$$

Then we get the following most general transformation law of the Lagrangian under Galilei symmetries

$$\mathcal{L}(gq, g\dot{q}) = \mathcal{L}(q, \dot{q}) + m\left(\frac{1}{2}v^2 \dot{t} - \vec{v} \cdot R\dot{\vec{r}}\right), \tag{35}$$

where gq is given by (32) and

$$g\dot{q} = (\dot{t},\ R\dot{\vec{r}} - \vec{v}\dot{t}). \tag{36}$$

Choosing $g = g_q^{-1} = (-t, -\vec{r}, 0, 1)$, we get

$$\mathcal{L}(q_0, \dot{q}) = \mathcal{L}(q, \dot{q}).$$

Therefore, $\mathcal{L}$ does not depend on $\vec{r}$ and t, as was assumed by Landau and Lifshitz. But now we have a rigorous justification why we can make such a choice without loss of generality in spite of quasi-invariance of the Lagrangian.

Hence $\mathcal{L}(q, \dot{q}) = \mathcal{L}(\dot{t}, \dot{\vec{r}})$ and we can rewrite (35) as follows

$$\mathcal{L}(\dot{t}, R\dot{\vec{r}} - \vec{v}\dot{t}) = \mathcal{L}(\dot{t}, \dot{\vec{r}}) + m\left(\frac{1}{2}v^2 \dot{t} - \vec{v} \cdot R\dot{\vec{r}}\right). \tag{37}$$

For $\vec{v} = R\dot{\vec{r}}/\dot{t}$, we get (note that $(R\dot{\vec{r}})^2 = \dot{\vec{r}}^2$)

$$\mathcal{L}(\dot{t}, 0) = \mathcal{L}(\dot{t}, \dot{\vec{r}}) - \frac{m}{2}\frac{\dot{\vec{r}}^2}{\dot{t}}. \tag{38}$$

The Lagrangian $\mathcal{L}$ is a homogeneous function of first degree in the $\dot{t}$ and $\dot{\vec{r}}$ derivatives (see (4)). Therefore, $\mathcal{L}(\dot{t}, 0) = E_0\dot{t}$, where E_0 is some arbitrary constant, and from (38) we get the most general form (up to equivalence) of the Lagrangian compatible to the Galilei symmetry

$$\mathcal{L}(\dot{t}, \dot{\vec{r}}) = E_0\dot{t} + \frac{m}{2}\frac{\dot{\vec{r}}^2}{\dot{t}}. \tag{39}$$

Taking $s = t$, so that $\dot{t} = 1$ and $\dot{\vec{r}}$ is the particle velocity, we recover the standard result

$$\mathcal{L}(\dot{t}, \dot{\vec{r}}) = E_0 + \frac{m}{2}\left(\frac{d\vec{r}}{dt}\right)^2. \tag{40}$$

The rest energy E_0 which appears in (40) has no real significance in classical mechanics and can be omitted from (40) without changing equations of motions. Note however, that the way how this rest energy was introduced in the theory indicates that E_0 has no relation with the mass m of the particle. In non-relativistic physics (or more precisely, in Galilei invariant theory), E_0 and m are two unrelated constants characterizing the particle (E_0 being insignificant as far as the classical mechanics is concerned).

Only in relativity E_0 and m proved to be related by Einstein's famous $E_0 = mc^2$ formula, as (11) indicates (in fact, it was Max Laue who produced the first general correct proof of this relation in 1911 for arbitrary closed static systems, generalized by Felix Klein in 1918 for arbitrary closed time-dependent systems [34]).

Interestingly, we can get a strictly invariant Lagrangian if we enlarge the configuration space of the system by introducing just one additional real variable θ. Indeed, let us consider the Lagrangian

$$\tilde{\mathcal{L}} = \mathcal{L} - \dot{\theta}. \tag{41}$$

It is obviously invariant upon transformations

$$q' = gq, \quad \theta' = \theta + \alpha(g; q), \tag{42}$$

because

$$\mathcal{L}(q', \dot{q}') = \mathcal{L}(q, \dot{q}) + \frac{d}{ds}\alpha(g; q).$$

Unfortunately, transformations (42) do not form a group.

$$g_1[g_2(q, \theta)] = (g_1 g_2 q,\, \theta + \alpha(g_2; q) + \alpha(g_1; g_2 q)) =$$

$$(g_1 g_2 q,\, \theta + \alpha(g_1 g_2; q) + \xi(g_1, g_2)) \neq (g_1 g_2)(q, \theta) = (g_1 g_2 q,\, \theta + \alpha(g_2; q)).$$

As we see, the presence of the $\xi(g_1, g_2)$ cocycle makes it impossible to define the multiplication law $g_1 \odot g_2$ because $g_1[g_2(q, \theta)] \neq (g_1 g_2)(q, \theta)$.

However, there is a simple way out. The Lagrangian (41) is obviously invariant under transformations

$$q' = q, \quad \theta' = \theta + \Theta_1, \tag{43}$$

with some constant Θ_1. Let us combine the transformations (42) and (43) in the following way

$$(g, \Theta)(q, \theta) = (gq,\, \theta + \Theta + \alpha(g; q)). \tag{44}$$

Then $g_1[g_2(q, \theta)] = (g_1 g_2)(q, \theta)$ condition requires the following multiplication law

$$(g_1, \Theta_1) \odot (g_2, \Theta_2) = (g_1 g_2,\, \Theta_1 + \Theta_2 + \xi(g_1, g_2)). \tag{45}$$

It can be checked that in this case the cocycle condition

$$\xi(g_2, g_3) + \xi(g_1, g_2 g_3) = \xi(g_1, g_2) + \xi(g_1 g_2, g_3)$$

helps to ensure the associativity of the multiplication law (45) and the set of the (g, Θ) pairs, $\tilde{G}$, indeed form a group, the inverse element being

$$(g, \Theta)^{-1} = (g^{-1},\, -\Theta - \xi(g, g^{-1})).$$

Note that G is not a subgroup of $\tilde{G}$. Instead, G is isomorphic to the factor-group $\tilde{G}/\mathbb{R}$, where $\mathbb{R}$ is the Abelian group of transformations (43) (identical to the additive group of real numbers). It is said that $\tilde{G}$ constitutes a central extension of G. Central extensions play an important role in physics, especially in quantum physics [26, 35].

4 Mass and quantum theory

Although the classical theory, considered above, is completely sufficient to demonstrate our main point that the modern way of introducing mass makes it clear that it can not depend on velocity neither in Galilei nor in Poincaré invariant theory, the real basis of modern physics is quantum theory.

Through the Feynman path integral formalism, the quantum theory explains the appearance of the least action principle in classical theory [36]. Therefore, “there is no longer any need for the mystery that comes from trying to describe quantum behaviour as some strange approximation to the classical behaviour of waves and particles. Instead we turn the job of explaining around. We start from quantum behaviour and show how this explains classical behaviour” [37].

Unfortunately, modern education completely ignores this possibility and still speaks about “wave-particle duality” and “the complementarity principle” as a philosophical bases to fuse the two apparently contradictory ideas of classical particles and classical waves into a quantum concept. The following example [38] shows that such educational practice distorts the scientific integrity even of professional physicists. In his critique of the customary interpretation of quantum mechanics, Landé indicated the following paradox [39]. It seems the de Broglie relationship between the momentum and wavelength

$$p = \frac{h}{\lambda}$$

contradicts the principle of relativity because the momentum does depend on the choice of the reference frame while the wavelength does not. An amusing fact, according to Levý-Leblond [38], is that nowadays experimentalists in neutron optics have difficulties to grasp Landé's paradox. They know quite well, as firmly established experimental fact, that the wavelength λ is related to the momentum p by de Broglie's $\lambda = h/p$ and it *does change* from one inertial frame to the other. Why is then Landé claiming that λ is an invariant? Recovered from the "wave-particle duality" dichotomy by their professional experience, they missed to appreciate a simple fact, as the crux of the Landé's argument, that "in classical wave theory, λ indeed is an invariant: the crest-to-crest distance of sea waves is the same to an aircraft pilot and to a lighthouse keeper" [38]. They simply do not understand why this classical property of waves has any relation to quantum objects they study.

Although Landé's paradox has some interesting aspects related to classical special relativity [40], it is not a real paradox in quantum theory [41] and originates only if we still insist on the schizophrenic classical view of the quantum world that a quantum particle somehow manages to be simultaneously both a particle and a wave while in reality it is neither particle nor wave [42]. Simply "it must be realized today that this view of the quantum world, adapted as it was to its first explorations, is totally out-dated. In the past fifty years, we have accumulated sufficient familiarity, theoretical as well as experimental, with the quantum world to no longer look at it through classical glasses" [39].

Mathematically the Landé's paradox is the following. Wave function of a free non-relativistic particle (for simplicity, we will assume $\hbar = 1$) $\Psi(\vec{r}, t) = \exp\{i(Et - \vec{p}\cdot\vec{r})\}$ is not invariant under Galilei boosts

$$t' = t, \;\; \vec{r}\,' = \vec{r} - \vec{v}\,t, \;\; \vec{p}\,' = \vec{p} - m\,\vec{v}, \;\; E' = E - \vec{p}\cdot\vec{v} + \frac{mv^2}{2}.$$

That is $\Psi'(\vec{r}\,', t') \neq \Psi(\vec{r}, t)$, where $\Psi'(\vec{r}, t) = \exp\{i(E't - \vec{p}\,'\cdot\vec{r})\}$. This is of course true, but from the point of physics the strict invariance is not at all required. What is really required is

$$\Psi'(\vec{r}\,', t') = e^{i\alpha(g;\,\vec{r},t)}\,\Psi(\vec{r}, t). \tag{46}$$

The phase factor $\alpha(g;q)$ is not completely arbitrary. Let us make a closer look at conditions it must satisfy. Consider a sequence of Galilei boosts

$$q \to g_2 q \to g_1(g_2 q). \tag{47}$$

If we write (46) as

$$\Psi'(q) = e^{i\alpha(g;q)}\,\Psi(g^{-1}q), \tag{48}$$

then we get for this sequence (47):

$$\Psi''(q) = e^{i\alpha(g_1;q)}\,\Psi'(g_1^{-1}q) = e^{i[\alpha(g_1;q)+\alpha(g_2;g_1^{-1}q)]}\Psi(g_2^{-1}g_1^{-1}q). \tag{49}$$

We can state that the transformation (48) consistently realizes the invariance with regard to the Galilei boosts if the wave function (49) is physically indistinguishable from the wave function

$$\tilde{\Psi}''(q) = e^{i\alpha(g_1g_2;q)}\,\Psi((g_1g_2)^{-1}q), \tag{50}$$

associated to the direct $q \to (g_1g_2)q$ transition. Physical indistinguishability means that the transition amplitudes are the same:

$$\Psi''(q_2)[\Psi''(q_1)]^* = \tilde{\Psi}''(q_2)[\tilde{\Psi}''(q_1)]^* \tag{51}$$

for any two space-time points q_1 and q_2. Substituting (49) and (50) into (51), we get that the combination

$$\xi(g_1,g_2) = \alpha(g_2;g_1^{-1}q) - \alpha(g_1g_2;q) + \alpha(g_1;q) \tag{52}$$

must be independent on the space-time point q. Note that

$$\xi(g_1,g_2) = (\delta\alpha)(g_1,g_2;q).$$

Therefore $\xi(g_1,g_2)$ is locally trivial cocycle, but globally it is not necessarily trivial, that is, representable as the coboundary of a global cochain (this is just the case for the Galilei group, as we shall see soon).

The Landé paradox will be resolved if we show that de Broglie plane waves really induce a global cocycle (52). Let us write (48) for de Broglie plane waves

$$\exp\{-i(E't-\vec{p}\,'\cdot\vec{r})\} = e^{i\alpha(g;q)}\exp\{-i[Et-\vec{p}\cdot(\vec{r}+\vec{v}\,t)]\}.$$

Then we get

$$\alpha(g;q) = -\frac{mv^2}{2}\,t - m\vec{v}\cdot\vec{r}, \tag{53}$$

and we can check that

$$\xi(g_1,g_2)=0$$

for any two pure Galilei boosts g_1 and g_2. As we see, the phase factor (53) in (46) resolves the Landé paradox.

In fact $\xi(g_1,g_2)$ is the Bargmann cocycle (33) [43]. This can be shown as follows. Repeating the above reasoning for the general transformations from the Galilei group

$$\begin{aligned}
&gq = (t+\tau,\, R\vec{r}-\vec{v}\,t+\vec{a}),\\
&g^{-1}q = (t-\tau,\, R^{-1}(\vec{r}+\vec{v}\,t-\vec{a}-\vec{v}\,\tau)),\\
&(g_1g_2)q = (t+\tau_1+\tau_2,\, R_1R_2\vec{r}-(\vec{v}_1+R_1\vec{v}_2)t+\vec{a}_1+R_1\vec{a}_2-\vec{v}_1\tau_2),
\end{aligned} \tag{54}$$

we get

$$\alpha(g;q) = -\frac{mv^2}{2}\,(t-\tau) - m\vec{v}\cdot(\vec{r}-\vec{a}) - E'\tau + \vec{p}\,'\cdot\vec{a}. \tag{55}$$

Only the first two terms are relevant because the last two terms give a function of group parameters only (independent on $\vec{r}$ and t) and therefore lead to a globally trivial cocycle when substituted into (52). Keeping only the first two terms in (55) and taking into account (54), we get after some algebra

$$\alpha(g_2;g_1^{-1}q)-\alpha(g_1g_2;q)+\alpha(g_1;q) = \frac{m}{2}v_1^2\,\tau_2 - m\vec{v}_1\cdot R_1\vec{a}_2,$$

which is just the Bargmann cocycle (33).

The way we have obtained it shows that the Bargmann cocycle is locally trivial (is the coboundary of the local cochain $\alpha(g;q)$). However, it is globally nontrivial. Indeed, any globally trivial cocycle, having the form $\beta(g_2)-\beta(g_1g_2)+\beta(g_1)$, is symmetric in g_1 and g_2 on Abelian subgroups. It follows from (54) that elements of the form $(0, \vec{a}, \vec{v}, 1)$ (space translations and Galilean boosts) form an Abelian subgroup:

$$(0, \vec{a}_1, \vec{v}_1, 1) \cdot (0, \vec{a}_2, \vec{v}_2, 1) = (0, \vec{a}_1 + \vec{a}_2, \vec{v}_1 + \vec{v}_2, 1).$$

However, the Bargmann cocycle remains asymmetric on this Abelian subgroup, $\xi(g_1, g_2) = -m\vec{v}_1 \cdot \vec{a}_2$, and, therefore, it can not be a trivial cocycle. Moreover, the same argument indicates that different values of mass define not equivalent Bargmann cocycles because their difference, being asymmetric on the Abelian subgroup of space translations and Galilean boosts, is not a trivial cocycle. As we see, in non-relativistic physics, the mass of the particle has a cohomological origin, it parametrizes the central extensions of the Galilei group.

In non-relativistic physics, the mass is a primary concept and it is impossible to explain why only some central extensions of the Galilei group are realized as elementary particles. Relativity brings a big change in the conceptual status of mass. Einstein's $E_0 = mc^2$ "suggests the possibility of explaining mass in terms of energy" [44]. In fact, Quantum Chromodynamics already explains the origin of mass of most constituents of ordinary matter [45]. However, "our understanding of the origin of mass is by no means complete. We have achieved a beautiful and profound understanding of the origin of most of the mass of ordinary matter, but not of all of it. The value of the electron mass, in particular, remains deeply mysterious even in our most advanced speculations about unification and string theory. And ordinary matter, we have recently learned, supplies only a small fraction of mass in the Universe as a whole. More beautiful and profound revelations surely await discovery. We continue to search for concepts and theories that will allow us to understand the origin of mass in all its forms, by unveiling more of Nature's hidden symmetries" [44].

5 Concluding remarks

V. A. Fock once remarked that "the physics is essentially a simple science. The main problem in it is to understand which symbol means what" [46]. As we have seen above, the meaning of the symbol m which enters in Newton's $\vec{F} = m\vec{a}$ equation is more profound than the primary Newtonian "measure of inertia". Unfortunately, the modern education ignores all the twentieth century's achievements in deciphering this symbol and bases its exposition on the classical Newtonian physics as it was formed at the end of the nineteenth century, with only fragmentary and eclectic inclusions from the modern physics.

Not that classical physics is wrong and useless. Classical physics is a great achievement of humankind and fully deserves our admiration. At the end of their excellent book on classical mechanics [47], Sudarshan and Mukunda wrote "Classical mechanics is an eternal discipline, where harmony abounds. It is beautiful in the true sense of the term. It is new every time we grow and look at it anew. If we have conveyed a sense of awe and adoration to this eternal beauty, our labor is worthwhile".

But all the sparkling beauty of Classical Physics can manifest itself only when it is placed in a right framework of modern ideas. Archaic notions and concepts in education, like relativistic mass, not only hinder understanding of modern physics but also they make it impossible to truly appreciate the meaning of classical ideas and the context under which classical ideas are completely sound and operational.

Of course we are not talking about the terminology. If you like to have a special name for the combination $m\gamma$, or you feel that the relativistic mass will help a Newtonian intuition of your students to better understand some relativistic circumstances, no problem, go ahead and use it. It is the philosophy of teaching which is at stake. Let me try to explain by allegory.

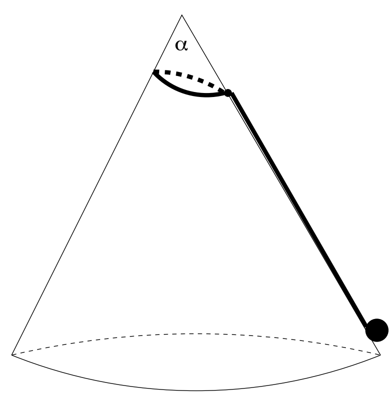

Figure 2: A student as a mountain climber.

Scientific progress needs an educated society and society has its special means to promote its members to higher level of education. A student under his/her search of higher education is like a mountain climber who wishes to climb up a frictionless conical mountain (see the figure. This nice mountain climber problem is from David Morin's superb textbook of introductory classical mechanics [48]). The aim of teaching was to give the student a lasso (a rope with a loop) and explain him/her how to use it. After sufficient teaching the student throws the lasso over the top of the mountain and climbs it up along the rope.

At the beginning the education was very much an elitist affair and students were given elitist deluxe lasso (see the figure).

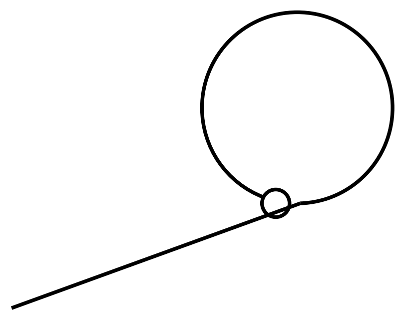

Figure 3: Deluxe lasso of nineteenth century education.

Deluxe lasso was quite effective till twentieth century and with it many famous scientists climbed up the icy mountain of contemporary science and furnished the glorious building of classical physics at the end of the nineteenth century. Educational standards were quite high, sometimes even unreasonably high and competitive. A good example is Cambridge Mathematical Tripos examinations, "a high speed marathon whose like has never been seen before or since" [49]. The examination lasted eight days and total number of questions was about two hundred. The results of examination were the subject of a great deal of public attention. The first man was called the Senior Wrangler followed by other Wranglers, the candidates awarded a first-class degree. Next came Senior Optimes, the candidates awarded a second-class degree, followed by Junior Optimes, the third class men. The procedure was quite cruel, some kind of Darwinian natural selection at work, as the following examples illustrate.

C. T. Simpson, the Second Wrangler in 1842, had worked twenty hours a day for a whole week before the examination and during the Tripos he almost broke down from overexertion [50].

James Wilson, the Senior Wrangler in 1859, experienced a severe mental and physical breakdown immediately after the examination. It took three months him to recover from the illness and after the recovery he found that he had forgotten utterly all the mathematics that he had learned at Cambridge apart from elementary algebra and Euclid [50, 51].

No wonder that the deluxe lasso became more and more ineffective at the beginning of twentieth century with its drastic change both in society and science. Then the cheap democratic lasso was invented (see the figure).

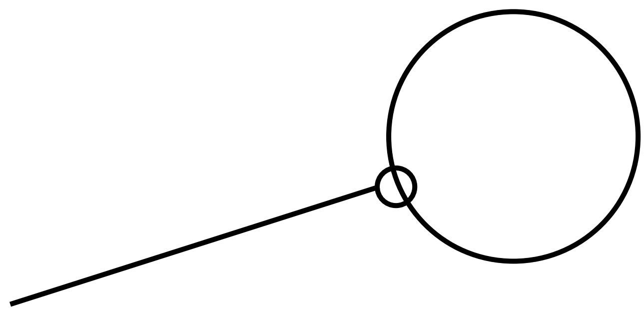

Figure 4: Democratic lasso of twentieth century education.

For decades the democratic lasso worked quite well and today we have the Standard Model and Large Hadron Collider. However, it seems even the democratic lasso becomes ineffective in our postmodern society. In fact, a gradual decrease of culture and intelligence is a long standing problem. Already Ludwig Wittgenstein, an Austrian philosopher, wrote in 1930 (cited in [52]): "I realize that the disappearance of a culture does not signify the disappearance of human value, but simply of certain means of expressing this value, yet the fact remains that I have no sympathy for the current of European civilization and do not understand its goals, if it has any. So I am really writing for people who are scattered throughout the corners of the globe".

It will be helpful to understand why lassos cease to be effective. It is just a simple mathematical exercise after the main physical essence of the problem is grasped [48]. The key to this problem is to realize that in the absence of friction the rope's tension ensures that the path of the lasso's loop on the cone's surface must be a geodesic (a nice discussion of geodesics on the cone's surface can be found in [53]). The cone's surface is flat. Let us cut it along a straight line joining the peak and the knot P of the lasso, and roll the cone flat onto a plane. We will get a sector of a circle and the lasso's loop will be represented by a straight line PP on this sector (after the cutting and rolling, the point P will appear at both sides of the involute of the cone at equal distances from the tip).

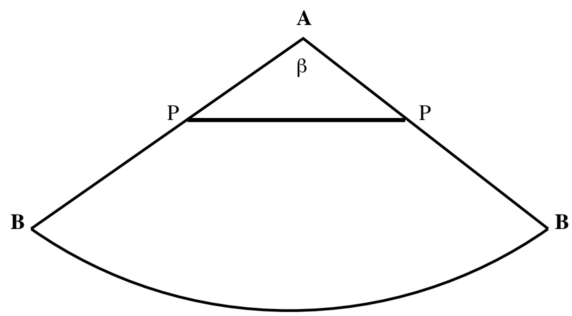

Figure 5: The involute of the cone and the path PP of the lasso's loop on it.

The deluxe lasso is one continuous piece of rope. Therefore, at the knot all three pieces of the rope will have the same tension and we need the angles between them to be 120° for the tensions to balance themselves. Hence the angle between PP and PB is 120°, the triangle APP is equilateral and $\beta = 60°$. What this means for the angle α of the cone? Let $AB = l$, then the radii of the cone's base is $r = l \sin \alpha/2$. We have two expressions for the arc-length L of BB. From the one hand, $L = \beta l$. On the other hand, $L = 2\pi r = 2\pi l \sin \alpha/2$ is the circumference of the cone's base. Equating these two expressions, we get

$$\sin \frac{\alpha}{2} = \frac{\beta}{2\pi}. \tag{56}$$

If $\beta = \pi/3$, then $\sin(\alpha/2) = 1/6$ and $\alpha \approx 19°$. Therefore, only in the special circumstances of the nineteenth century can the deluxe lasso work. It becomes useless then these circumstances change.

For the democratic lasso the angle between PP and PB is no longer constrained and it will always work unless the involute of the cone is greater than a semicircle. $\beta < 180°$ implies $\sin(\alpha/2) < 1/2$ and $\alpha < 60°$. This explains why the democratic lasso worked quite well in spite of changing conditions during the twentieth century.

But now, I'm afraid, the conditions both in society and science changed so drastically that we have $\alpha > 60°$. Therefore, the old lasso based education practise will no longer work irrespective how hard we try.

Modern education can no longer be based on Newton's laws and Newtonian concepts as primary building blocks. The progress in science was too great. Quantum mechanics and special relativity are cornerstones of modern physics. It is of crucial importance the modern education to be based on basic principles of these disciplines from the very beginning. Newton's laws and Newtonian concepts should be introduced as derivatives from these more profound theories, as they really are, and the limitations of the Newtonian concepts must be clearly stressed.

Without such a deep modernization of education, we will face a risk that the number of quasi-educated people will increase while the number of really intelligent people will decrease. We are already watching an alarming proliferation of irrationality and ignorance worldwide. A very clear explanation why such a situation is dangerous was given by Carlo Maria Cipolla, an Italian economic historian.

According to Cipolla [54], human beings fall into four basic categories: the helpless, the intelligent, the bandit and the stupid. If Tom takes an action and suffers a loss while producing a gain to Dick, Tom acted helplessly. If Tom takes an action by which he makes a gain while yielding a gain also to Dick, Tom acted intelligently. If Tom takes an action by which he makes a gain causing Dick a loss, Tom acted as a bandit. If Tom causes losses to Dick or to a group of persons while himself deriving no gain and even possibly incurring losses, he is a stupid person.

"Whether one considers classical, or medieval, or modern or contemporary times one is impressed by the fact that any country moving uphill has its unavoidable σ fraction of stupid people. However the country moving uphill also has an unusually high fraction of intelligent people who manage to keep the σ fraction at bay and at the same time produce enough gains for themselves and the other members of the community to make progress a certainty.

In a country which is moving downhill, the fraction of stupid people is still equal to σ; however in the remaining population one notices among those in power an alarming proliferation of the bandits with overtones of stupidity and among those not in power an equally alarming growth in the number of helpless individuals. Such change in the composition of the non-stupid population inevitably strengthens the destructive power of the σ fraction and makes decline a certainty. And the country goes to Hell" [54].

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